7 Brokers and bridges

7.1 Introduction

A person with many social relations has better chances of getting help or information. Therefore, social relations are called social capital, an asset which pays off. Network analysts, however, discovered that the kind of relation is important in addition to the sheer number of relations. Their general argument is that ‘strong’ relations with people who are themselves related yield less useful information than ‘weak’ relations with people who do not know one another. Having a lot of relations within a group exposes a person to the same information over and over again, whereas relations outside one’s group yield more diverse information which is worth passing on or retaining to make a profit.

As a consequence, we have to pay attention to the relations between a person’s contacts. A person who is connected to people who are themselves not directly connected has opportunities to mediate between them and profit from his or her mediation. The relations of this person bridge the structural holes between others. It is hypothesized that people and organizations who bridge structural holes between others have more control and perform better.

In this chapter, we will first discuss bridges at the level of the entire network (Section 7.3). Which relations (bridges) and which vertices (cut-vertices) are indispensable for the network to remain connected? If a network contains such relations and vertices, it contains ‘bottlenecks’ and the flow of information through the network is vulnerable. In the remaining sections, we will focus on brokerage at the level of individuals. Who is in the best position to profit from his or her social relations (Section 7.4) and how is this affected by group membership (Section 7.5)?

7.2 Example

The example of this chapter will show the importance of informal communication structures within a firm. In the wood-processing facility which already appeared in Chapter 6 a new management team proposed changes to the workers’ compensation package, which the workers did not accept. They started a strike, which lead to a negotiation stalemate. Then, management asked an outsider to analyze the communication structure among the employees because it felt that information about the proposed changes was not effectively communicated to all employees by the union negotiators.

The outside consultant asked all employees to indicate the frequency in which they discussed the strike with each of their colleagues on a 5-point scale, ranging from ‘almost never’ (less then once per week) to ‘very often’ (several times per
day). The consultant used three as a cut-off value. If at least one of two persons indicated that they discussed work with a frequency of three or more, a line between them was added to the informal communication network (Strike.net).

The network displays fairly stringent demarcations between groups defined on age and language [Figure 1]. The Spanish-speaking young employees, who are of age 30 or younger, (class one in the partition Strike_groups.clu, yellow vertices in Figure 1) are almost disconnected from the English-speaking young employees (class 2, green vertices), who communicate with no more than two of the older English-speaking employees (38 years or more, class 3: red vertices). These divisions mirror the homophily principle discussed in Chapter 3: people tend to relate to similar people.

All ties between groups have special backgrounds. Among the Hispanics, Alejandro is most proficient in English and Bob speaks some Spanish, which explains their tie. Bob owes Norm for getting his job and probably because of this, they developed a friendship relation. Finally, Ozzie is the father of Karl.

Figure 1 - Communication network of striking employees.

Sam and Wendle are the union negotiators (the boxes in Figure 1). They were responsible for explaining the new program proposed by the managers. When the informal communication structure among employees was reported to the management, they approached two of the other employees directly (guess who?) to explain the reforms to them personally. Then, they gave them some time to discuss the plans with their colleagues. Within two days, the young and old employees were willing to strike a deal with the management and they persuaded the union representatives to reopen negotiations. Soon, the labor dispute was reconciled and the strike ended.

7.3 Bridges and bi-components

The example shows the importance of social ties to the diffusion of information. Information is the key to the exploitation of social relations as social capital. Social ties offer access to information, which can be used to reduce uncertainty.
and risk, and to create trust, for instance, when information is confirmed from several sources. People on crucial positions in the information network may also spread or retain information strategically because they have control over the diffusion of information.

In a social system, for instance, an organization, the overall structure of informal ties is relevant to the diffusion of information. Can information reach all members of the organization or is it more likely to circulate in one segment of the network? Are there any bottlenecks which are vital to the flow of information, which may prohibit the spread of information because of information overload or because people pursue their private, strategic goals?

In Figure 1, the tie between Alejandro and Bob is clearly a bottleneck because it is the only channel for information exchange between the Hispanic employees and all other employees. Removing this single line will cut off the Hispanics from information circulating among the other employees. Formally, this line is a bridge in the network because its removal creates a new component, which is isolated from other components. The strike network consists of one component (recall that a component is a maximal connected subnetwork, see Chapter 3), so information may travel to each employee via social ties. When you remove the line between Alejandro and Bob, you disconnect the Hispanic workers from the communication network, so they become a component on their own.

A bridge is a line whose removal increases the number of components in the network.

Note that there is one more bridge in the information network of striking employees: the tie between Frank and Gill. If you remove this tie, Frank becomes an isolate. Since an isolate is a component, the network consists of two components after removing Frank’s tie with Gill.

The removal of a line may annul the connectedness of a network or component but the deletion of a vertex may have the same effect, because you remove the lines which are incident with the deleted vertex. After all, you cannot have a line with a single endpoint! When Bob refuses to discuss the strike anymore, he is lost to the communication network and all of his ties disappear, including the bridge to Alejandro. Therefore, Bob is a cut-vertex: its deletion disconnects the network or it disconnects a component of the network. Just like a bridge, a cut-vertex is crucial to the flow of information in a network. It is a ‘bottleneck’ in the network which controls the flow from one part to another part of the network. Norm, for example, is indispensable for exchanging information between the older and younger employees.

Deleting a vertex from a network means that the vertex and all lines incident with this vertex are removed from the network. A cut-vertex is a vertex whose deletion increases the number of components in the network.
Figure 2 - Bi-components and cut-vertices in the strike network.

In Figure 2, all cut-vertices are brown. Note that vertices incident with a bridge may or may not be cut-vertices. Alejandro and Bob are cut-vertices, but Frank is not, because removal of Frank and his bridge to Gill does not increase the number of components.

Now that we have defined cut-vertices, it is easy to define sections of a network which are relatively invulnerable to the withdrawal or manipulation of a single vertex, namely a bi-component. A bi-component is simply a component - a maximal connected subnetwork - without a cut-vertex. In a bi-component, no person can control the information flow between two other persons completely because there is always an alternative path which information may follow. In a bi-component, each person receives information from at least two sources (in an undirected network), so s/he may check the information.

A bi-component is a component which does not contain a cut-vertex.

Figure 2 shows the cut-vertices, which are brown, in the strike network. A cut-vertex always belongs to two or more bi-components. Probably, this sounds weird to you: didn’t we define a bi-component as a component without cut-vertices, then how can a cut-vertex belong to two or more bi-components? The answer is that a bi-component does not contain cut-vertices if you look at the bi-component only and ignore the rest of the network. Concentrate on the Hispanic employees, for instance: if you remove Alejandro, the other three Hispanics remain connected into one component, so the removal of Alejandro does not increase the number of components among the Hispanic employees.

Looking at the entire network, however, Alejandro is a cut-vertex because he belongs to the Hispanic (red) bi-component and to a small bi-component with Bob: the two bi-components overlap. A network usually consists of overlapping bi-components.

Knowing this, you may now identify the six bi-components in the strike network. Five bi-components are identified by the color of their vertices: yellow, green, pink, red, and white. Keep in mind that the (brown) cut-vertices which are
incident to the vertices of a bi-component also belong to that bi-component. The bridge between Bob and Alejandro is the sixth bi-component, which consists only of cut-vertices, so it cannot be identified by vertex color.

The two union representatives among the employees, Wendle and Sam, are part of one small bi-component which is connected to the bi-component of older employees by Sam. So we may say that Sam controls the information exchange between the union representatives and all other employees except Xavier. If Sam does not want to strike a deal with the management of the firm, he can manipulate the information to and from the other employees.

In numerous applications, it has been shown that people with strong ties are cliques or tend to develop into cliques, for example, family ties are usually strong in the sense that they are intense, and family relations display cliques: several or all members of a family maintain strong relations among themselves. As a consequence, family ties are not very useful in finding a new job because they relate you to people with whom you are already related. Usually, they do not supply information about new jobs of which you have not heard already. In contrast, superficial and irregular contacts such as former colleagues or acquaintances are better sources of information on new job opportunities. These weak ties are more likely to be bridges to distant information networks, hence the concept of “the strength of weak ties”, meaning that weak ties are often more important for the dispersion of information than strong ties. The strength of a tie may be taken as a proxy of its chances to be a network bridge.

This hypothesis could apply to the Spanish-speaking employees. Strong ethnic ties develop into a clique and the only non-ethnic tie (between Alejandro and Bob) is a bridge to the rest of the network. In this example, however, family ties connect employees outside cliques: Gill is Frank’s cousin and Ozzie is Karl’s father. We should note that the strength of weak ties depends on the situation: on the shop floor, family ties, which are usually considered strong, may fulfill the ‘bridging’ role of weak ties because family ties are uncommon and they will not develop into cliques within the firm (a firm is not the natural setting to raise a family).

Remember, however, that we only consider the stronger communication ties because irregular communication ties are disregarded (scores one and two on the 5-point scale) in this example. The strength of weak ties argument predicts that strong ties will constitute cliques, which is clearly the case. Perhaps, the weaker ties cross group boundaries more often. Note, that the strength of a tie may be defined in several ways, e.g., frequency versus social intensity, which is important to consider when you apply the strength of weak ties hypothesis.

**Application**

You may use the *Bi-Components* command in the *Net > Components* submenu for finding bi-components and cut-vertices in a network. When asked, choose two as the minimum size of a bi-component, unless your network contains many strings of bridges which you do not want to count as separate bi-components. If you
choose a larger minimum size, however, you must realize that you may not identify all cut-vertices, which are also called articulation vertices, in the network.

Pajek treats directed networks as if they were undirected, which means that it identifies weak components without cut-vertices in directed networks. If you symmetrize a directed network before you execute the Bi-Components command, you will obtain exactly the same results.

The output of the Bi-Components command consists of two partitions and a hierarchy. The first partition, which is most useful, indicates the number of the bi-component to which a vertex belongs. Vertices which do not belong to a bi-component, e.g., isolates, are collected in class 0 and vertices which belong to two or more bi-components - cut-vertices - are placed in class number 999998. This partition is used in Figure 2.

The second partition indicates the number of bi-components to which a vertex belongs: 0 for isolates, 1 for a vertex which belongs to exactly one bi-component, 2 for vertices which belong to two bi-components, etc. Finally, the hierarchy shows the bi-components to which each vertex belongs. Note that we need a hierarchy to store the bi-components because cut-vertices belong to two or more bi-components.

In an undirected network without multiple lines, such as our example, bridges are bi-components of size two (see Exercise 5). Therefore, you can easily find the bridges in the hierarchy of bi-components: open the Edit screen with the hierarchy of bi-components (see Figure 3) with the command File>Hierarchy>Edit or with the Edit button on the left of the hierarchy drop list. Figure 3 lists the six bi-components in the communication network among striking employees. The size of each bi-component is reported between brackets, so it is easy to find the two bridges in this example: bi-components four and six. Double-click (or single right mouse click) them to see their vertices.

Figure 3 - Hierarchy of bi-components in the strike network.

7.4 Ego-networks and constraint

In the previous section, we analyzed the structure of the entire network, which is a socio-centered approach. Now, we turn to the ego-network or ego-centered
approach: we focus on the position of one person in the network and his or her opportunities to broker or mediate between other people.

Let us first have a look at a triad, which consists of a focal person (ego), an alter, a third person, and the relations among them. The triad is the smallest network which contains more than two persons and it highlights the complexities of relations within a group. According to the sociologist Georg Simmel, a complete triad (A in Figure 4) reduces the individuality of its members. When three people are fully connected, they share norms and information, they create trust by feedback, and conflicts between two members may be resolved or moderated by the third person. In other words, complete connections between three persons make them behave as a group rather than as a set of individuals.

Figure 4 - Three connected triads.

In an undirected triad which is connected but incomplete, for instance, networks B and C in Figure 4, people are considered to be bound less by group norms. One person is in an advantageous, powerful position, because s/he may broker between the other two. The person in the middle (ego in B and alter in C, Figure 4) may profit from the competition between the other two, for example, ego negotiates the price of a good or service to be delivered by either alter or the third party in network B. Ego makes them compete, which would not be possible if alter and third would agree about a price among themselves. This is known as the tertius gaudens (“the third who benefits”) or the tertius strategy: induce and exploit competition or rivalry between the other two, who are not directly related. The absence of a relation between alter and the third party is known as a structural hole, which may be exploited by ego.

A more malicious variant is known as divide et impera or the divide and rule strategy, in which a person creates and exploits conflict between the other two in order to control both of them, for example, ego tells alter unpleasant things about the third party and the third party about alter, which results in hostility among them. This would not be possible if they could directly check the information and find out ego’s subversive strategy. Again, the structural hole allows ego to apply this strategy.

In both strategies, an individual’s advantage or power is based on his or her control over the spread of information, goods, or services, which stems from the structure of his or her network. We want to stress that brokerage is related to the absence of relations or ‘holes’ between neighbors, whereas we concentrated on the presence of relations in our chapters about cohesive subgroups in Part II.

The opportunities which a structural hole offers in an incomplete triad have a reverse side: they imply constraint in a complete triad. A complete triad is not just
a triad without opportunities because it has no structural holes. No, the situation is even worse from the perspective of brokerage, because you cannot withdraw from any of these unrewarding relations without creating a structural hole around yourself. In network A (Figure 4), ego is more or less obliged to maintain both relations, because if ego stops the relation with one (e.g., with the third in A, so triad C evolves), there is a structural hole around ego, which alter may take advantage of.

The ego-network of a vertex contains this vertex, its neighbors and all lines among the selected vertices.

Now, let us focus on the ego-network, which consists of ego, ego’s neighbors, and the relations among them. Alejandro’s ego-network is displayed in Figure 5. Note that this network fully contains the ego-networks of Carlos, Domingo, and Eduardo. The ego-network of a person contains all triads which include this person, so we can analyze it as a set of triads. For each triad, we can determine whether it constrains ego or whether it contains a structural hole which ego may exploit, for instance, Alejandro (ego) has an opportunity to broker between Bob (alter) and Domingo (third) because Bob and Domingo are not directly connected. For the same reason, Alejandro may broker between Bob and Carlos or Eduardo. There are three triads in Alejandro’s ego-network which give him an opportunity to broker for Bob.

Figure 5 - Alejandro’s ego-network.

In a similar way, we can compute the constraint on Alejandro which is exercised by his relation with Bob: the number of complete triads containing Alejandro, Bob and another neighbor of Alejandro. Since no other neighbor of Alejandro is directly connected to Bob, there is no constraint on Alejandro because of his relation with Bob. A low constraint indicates many structural holes, which may be exploited. In contrast, the constraint on Alejandro’s relations with Carlos, Domingo, and Eduardo is very high because these relations are involved in three complete triads. When Alejandro withdraws from any of these relations, they may start brokering for him.

The higher the constraint, the fewer the opportunities to broker, and the more dangerous it is to withdraw from a relation. This constraint is known as the dyadic constraint associated with a relation from ego’s point of view. Note that the constraint of a relation on ego may differ from the constraint experienced by alter on the same relation. The relation between Alejandro and Carlos, for
instance, is more constrained for Carlos than for Alejandro, because all Carlos’
triads are complete.

In our discussion of structural holes and constraint, something is still missing:
we ought to take into account the importance of a tie to a person. If a tie is very
cheap in terms of investment (money, network time, and energy), it is not really a
problem to be obliged to maintain it. If a tie is just one among many (low
exclusivity), ego does not depend on this tie much and it is no big deal if alter
threatens to break it. Besides, if the tie between alter and the third party is not
important to them, it may function like an absent tie, which can be exploited.

The **proportional strength** of a tie with respect to all ties of a person is a
simple indicator of the importance or exclusivity of a tie. It is computed as the
value of the line(s) representing a tie, divided by the sum of the values of all lines
incident with a person. If line values express costs, time, or energy, the
proportional strength of a tie is the portion of an actor’s total expenditure which
is ‘invested’ in the ties with an alter. Just like dyadic constraint, it makes a
difference from which standpoint you look at the network, for instance, the tie
between Alejandro and Bob is one out of Alejandro’s four relations (0.25) but it
is only one of Bob’s seven relations (0.14), so the proportional strength of a tie
must be represented by a directed network. Note that the original
network may contain multiple lines, directed and undirected lines, and line
values, but the network with proportional strength relations is always simple
directed and contains only bi-directed arcs.

![Diagram showing proportional strength relations around Alejandro](Image)

**Figure 6** - Proportional strength of ties around Alejandro.

The **dyadic constraint** on vertex \( u \) exercised by a relation between vertices \( u \) and
\( v \) is the extent to which \( u \) has more and stronger relations with neighbors who are
strongly connected with vertex \( v \).

This definition describes the ideas behind dyadic constraint rather than the exact
computation. For those interested in the exact computation: add the proportional
strength of the relation from ego to alter (investment of ego in alter) to the
products of the proportional strength of the two arcs in each path from ego to
alter via another neighbor of ego and take the square of this sum.
The constraint on Alejandro attached to his relation with Eduardo is equal to the square of the following sum: 0.25 (Alejandro’s investment in Eduardo), plus 0.25*0.33 (Alejandro’s tie to Carlos times Carlos’ tie to Eduardo), plus 0.25*0.33 (idem via Domingo). All numbers are proportional strengths which can be read from Figure 6. The sum is 0.415 and the square of this sum is 0.17 (see Figure 7). As you may have expected, the constraint on Alejandro which is attached to his ties with Domingo or Carlos is also 0.17 and the constraint on his tie with Bob is just the square of the proportional strength of this tie (0.0625) because there are no indirect paths from Alejandro to Bob in Alejandro’s ego-network.

We may conclude that the constraint on Alejandro’s tie with Bob is about one third of the constraint of his ties within the Hispanic cluster. Clearly, the structural holes in Alejandro’s network are attached to his link with Bob: he is able to play the tertius gaudens role between Bob and the other Hispanics because he may act as a representative of the Hispanics towards Bob.

**Figure 7 - Constraints on Alejandro.**

If we have the dyadic constraint on all relations of a person, we can simply add them to obtain the **aggregate constraint** on this person. The aggregate constraint is a nonnegative number which is usually between 0 and 1 but it can be more than one. The aggregate constraint on Alejandro, for instance, is $0.174 + 0.174 + 0.174 + 0.063 = 0.585$. The higher the aggregate constraint, the less ‘freedom’ a person has to withdraw from existing relations or to exploit structural holes.

People or organizations with low aggregate constraint are hypothesized to perform better. It has been shown that employees with low constraint in an organization have more successful careers and that business sectors with lower constraint on firms are more profitable. In general, researchers compare the constraint on an actor to one or more indicators of its (economic) success. In our example, this could be the success in resolving the conflict between employees and management or personal influence on the conditions specified in the final agreement. Bob and Norm negotiated the proposal with the management before they called in the union representatives, so they may have been successful in changing the conditions according to their interests.

**Application**

In Pajek, one command computes the proportional strength of relations, the dyadic constraint, and the aggregate constraint for all vertices in a network. This command is aptly called **Structural Holes** and you can find it in the *Net>Vector* submenu. The proportional strength of relations is output as a new network and
so is dyadic constraint. In these networks, the line values express the strength and constraint on relations respectively. Note that these networks are always directed and that all arcs are reciprocated, no matter whether the original network is directed or undirected, valued or unvalued, whether it contains multiple lines and loops or not.

There is an easy way to visualize the structural holes in a network. Take the network of dyadic constraint and energize it using line values as similarities (option Options > Values of Lines > Similarities in the Draw screen). Now, vertices which are tied by relations of high constraint are drawn closely together, whereas relations of low constraint are long, so they create a lot of space between the vertices which looks like a hole (Figure 8).

![Figure 8 - Energized constraint network.](image)

Aggregate constraint is output as a vector. You may inspect this vector in the usual ways with the Info > Vector command or by editing it. If you want the size of the vertices to represent their aggregate constraint in the Draw screen, we advise to multiply the vector by 10 (command Vector > Transform > Multiply by) or use the Autosize option in the Options > Size of Vertices submenu of the Draw screen, otherwise the vertices are drawn too small.

### 7.5 Affiliations and brokerage roles

Group affiliation is often important in brokerage processes. A union representative mediates between the management and the workers. He can negotiate with one manager or another and he can choose whom of his colleagues to consult. To some extent, his contacts are replaceable by someone else from the same group. Moreover, the union representative himself must belong to a particular group, namely the workers. In our example, the union representatives Sam and Wendle are a subgroup of workers and the management is supposed to negotiate with them. This restricts the managers’ choice of negotiation partners.
enormously, so they have little opportunity to play off one negotiator or worker against another. In this case, the opportunity to broker depends not only on the position of people in the network but also on their group affiliations.

The easier it is to replace your contact by someone else from his or her group, the stronger your position is to negotiate and the higher the chance of striking a good deal or of getting things done your way. The replacement does not have to be one of your present contacts, it may be someone outside of your present egonetwork whom you may consider to include in it at the expense of someone else. Is there someone else in the group of your contact who is at least as central as your contact but who is not directly linked to your contact so including him in your ego-network would create a structural hole between your present contact and the new contact? Such a structural hole is called a **secondary structural hole**.

Let us illustrate this with an example. Suppose Alejandro wants to play a divide and rule strategy against Bob because he feels too constrained by him. Bob is in a good structural position to negotiate on behalf of the English-speaking young employees because he is directly connected to most members of this group. It is very difficult to spread discord among members of his group by spreading rumors about Bob, because the other members of the group are likely to inform Bob when they maintain direct relations. Alejandro’s best choice seems to be Gill, because he is not directly connected to Bob and it is very likely that Frank will team up with him, after which Gill may try to play Ike versus Hall and John, although these colleagues are constrained by Bob. Frank and Karl, the only other English-speaking young employees who are not directly related to Bob, are less suited as an alternative to Bob because they are less central in the group.

Since secondary structural holes concern the relations within one group, namely the opportunities to exploit structural holes within that group, the aggregate constraint within a group seems to be a useful indicator of whom to contact and persuade as an alternative to your present contact in the group: the person who is least constrained and who is not constrained by your present contact because he is not directly tied to him or her (see Figure 11 in the Application Section).

**Figure 9** - Five brokerage roles of actor $v$.

For another approach to brokerage and affiliations, we have to turn to triads again. A triad in which person $v$ mediates transactions between persons $u$ and $w$ can display five different patterns of group affiliations, which are indicated by vertex color as well as contours in Figure 9. Each pattern is known as a brokerage role. Research into brokerage roles is concerned with describing the types of
brokerage roles which dominate a transactional or exchange network. In addition, individual positions within the network may be characterized by the dominant type of brokerage role and hypotheses may be tested about the personal characteristics of individuals with certain types of brokerage roles.

Two brokerage roles involve mediation between members of one group. In the first role, the mediator is also a member of the group. This is known as the coordinator role. In the second role, two members of a group use a mediator from outside, an itinerant broker. The other three brokerage roles describe mediation between members of different groups. In one role, the mediator acts as a representative of his group because he regulates the flow of information or goods from his or her own group. In another role, the mediator is a gatekeeper, who regulates the flow of information or goods to his or her group. Finally, the liaison is a person who mediates between members of different groups but who does not belong to these groups himself or herself.

The five types of brokerage roles have been conceived for directed networks, namely transaction networks. Note, however, that the direction of relations is only needed to distinguish between the representative and the gatekeeper. The other brokerage roles are also apparent in undirected relations, so we can apply the brokerage roles to undirected networks if we do not distinguish between representatives and gatekeepers. In an undirected network, each representative is also a gatekeeper and vice versa.

Now, let us have a look at the brokerage roles in the strike network. We use the groups according to language and age (see Figure 1) and we assume that a line is equivalent to a bi-directional arc: discussing work implies the possibility to spread and receive information. Employees who are isolated or whose relations are contained within a clique, e.g., Carlos, Domingo, and Eduardo but also Wendle and Xavier, have no opportunity to mediate because all of their contacts are directly connected. As a result, none of the brokerage roles apply to them.

Most of the other employees have relations only within their own group, so they can only play the coordinator role. In the network, brokerage is clearly dominated by the coordinator role. It is easy to see that Alejandro, Bob, Norm, and Ozzie are the only employees who also have other types of brokerage roles because they are the only ones who are connected to members of different groups.

Figure 10 - Bob’s ego-network.
Let us have a closer look at Bob (Figure 10), who combines several types of brokerage roles. There are several structural holes among Bob’s relations within the group of English-speaking young employees, e.g., between Ike and Mike on the one hand, and Hal, John, and Lanny on the other hand. To them, Bob plays the coordinator role. In addition, Bob bridges many structural holes between his group of English-speaking young employees and the Hispanic workers or the older employees. For information about his group, Bob is a representative and for information flowing towards members of his group, he is a gatekeeper. Finally, he may mediate between Alejandro and Norm, that is, between the Hispanics and the older workers. In this role, he is a liaison. The only brokerage role which Bob cannot play given the relations in the network, is the role of an itinerant broker because he does not have relations with two or more members of any group other than his own. Actually, none of the employees can play this role - this role is absent in the strike network.

Bob was the first employee whom the management contacted directly. Perhaps, this was justified not only by the amount of structural holes in his ego-network but also by the variety of brokerage roles which Bob may play.

Application

Secondary structural holes are related to constraint within a group, so we may delete the relations between groups and calculate the constraint within each group. If there is another member of the group with equal or less constraint than the one you are already connected to, you may play him off against your present contact provided that they are not strongly and directly connected. In a similar manner, you may evaluate your position within your own group to see whether you may easily be replaced by someone else.

The detection of secondary structural holes consists of two steps. In the first step, we delete the lines between groups. Since the groups are defined as classes in a partition (in our example Strike_groups.clu), we must make sure that this partition is selected in the partitions drop list. Then we can remove the lines between clusters with the command Operations>Transform>Remove Lines>Between Clusters.

In the second step, we apply the Structural Holes command to the network without lines between clusters to obtain the constraint of vertices within their groups (Figure 11). Now, we can see that Gill is even less constrained within the class of young English-speaking employees than Bob. Since there is no direct relation between Gill and Bob, Gill seems to be a good candidate to be played off against Bob. In the Hispanic group, there is no real alternative to Alejandro because he is directly connected to all others. Among the older employees, Norm is clearly less constrained than any other employee, so there is no good alternative in this group. Judging from their structural positions and ignoring their linguistic abilities or special relationships, we conclude that Norm and Alejandro
are less likely to be replaced as representatives or gatekeepers of their groups than Bob because there is a good alternative to Bob only.

![Diagram showing social network analysis](image)

**Figure 11** - Constraint inside groups.

Pajek contains a command which counts the brokerage roles in a network. Make sure that the network and the appropriate partition are selected in the drop lists of the Main screen. Then, execute the command `Operations > Brokerage Roles` to obtain five partitions, one for each brokerage role. The class number of a vertex in a partition specifies the number of incomplete triads in which this vertex plays the corresponding brokerage role. A frequency table of a partition is obtained in the usual way (`Info > Partition`). Table 1 shows the results for the coordinator role.

**Table 1** - Frequency tabulation of coordinator roles in the strike network.

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<th>CumFreq</th>
<th>CumFreq%</th>
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</tr>
<tr>
<td>5</td>
<td>1</td>
<td>4.167</td>
<td>21</td>
<td>87.5000</td>
<td>Gill</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>4.167</td>
<td>22</td>
<td>91.6667</td>
<td>John</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>4.167</td>
<td>23</td>
<td>95.8333</td>
<td>Bob</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>4.167</td>
<td>24</td>
<td>100.0000</td>
<td>Norm</td>
</tr>
<tr>
<td>Sum</td>
<td>24</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We can see that 10 employees have no coordinator roles. The number of coordinator roles pro person is unevenly distributed: quite some employees have one or two coordinator roles whereas Norm has no less than nine coordinator roles. If you sum the roles (five employees with one coordinator role plus three with two roles, etc.), we count 46 coordinator roles. In the representative roles partition we count 21 roles and there is just one liaison role. As noted before, the
coordinator role occurs most frequently in this network because most employees have direct relations only within their own subgroup.

7.6 Summary

In a connected social network, information may reach anybody through their ties. Holes in this network, that is, absent ties, are obstacles to flows of information. Information is less likely to reach anybody easily in a connected network with large holes. In this chapter, we focus on the holes in a network. The information flow is especially vulnerable in networks with bridges and cut-vertices because the removal of a bridge or cut-vertex disconnects the network. Actors who are cut-vertices in a network control the flow of information from one part of the network to another. They may decide to retain information when it suits their personal purposes.

From the perspective of the communication system as a whole, bridges and cut-vertices are undesirable. From the individual point of view, however, being a cut-vertex is attractive because it offers opportunities for brokering information and for profiting from brokerage in one way or another. The advantage of the broker position in an ego-network, which is the network of one actor and its neighbors, is that you can play a tertius strategy: you can induce competition or conflict between neighbors who are not linked directly. The gap between the neighbors is called a structural hole and each structural hole represents an opportunity to broker.

There is drawback, however: you must avoid to be the object of a tertius strategy yourself. This implies that you cannot end relations to neighbors who are directly linked. This is called the constraint on your relation with a neighbor. The constraint on a relation is inversely related to the structural holes associated with it: low constraint means many structural holes and vice versa. The constraint on each relation as well as on each vertex in the network may be calculated to find the segments which have most opportunities to broker, which are hypothesized to be more successful and profitable.

In many social contexts, brokerage is connected to group affiliations and the people involved can only be replaced by other persons from their groups. Threatening a contact to replace him or her by another contact is also a tertius strategy. This strategy is only successful if there is a good alternative to the present contact in the group, someone who is also quite central in the group but who is not directly linked to the present contact so the (secondary) structural hole between them can be exploited.

When we consider brokerage in the context of group membership, there are five brokerage roles. We can characterize a network or an actor in a network by the kinds of brokerage roles which occur. The brokerage signature of a network or actor can be compared to other characteristics to determine whether certain types of actors or types of social relations develop particular brokerage roles.
7.7 Exercises

1. List the bridges in the network depicted below.

2. How many bi-components and cut-vertices does the network of exercise 1 contain?
   a. Three bi-components and three cut-vertices.
   b. Three bi-components and six cut-vertices.
   c. Six bi-components and three cut-vertices.
   d. Six bi-components and six cut-vertices.

3. Let the network of Exercise 1 represent the communication network within an organization. If you would like to reduce the power of cut-vertices to control the flow of information, which pair of vertices would you urge to establish a communication relation? Justify your answer.

4. Which of the following statements is correct?
   a. Vertices incident with a bridge are cut-vertices if and only if they have two or more neighbors.
   b. Vertices which are part of a bi-component cannot be cut-vertices.
   c. A bi-component is a subnetwork without cut-vertex.
   d. If there are two paths between all pairs of vertices in a component, this component is a bi-component.

5. Use the definitions of a bridge, a cut-vertex, and a bi-component to explain that two vertices connected by a bridge are a bi-component of size two and that each bi-component of size two contains a bridge in a simple undirected network.

6. Which of the following statements about the strength of weak ties hypothesis is correct?
   a. Strong family ties cannot be bridges in a communication network.
   b. A tie is weak if and only if it is a bridge in a communication network.
c. In general, weak ties are more likely to be bridges in a communication network.

d. A tie is strong if and only if it is part of a clique in a communication network.

7. In the network of Exercise 8, which vertex is least constrained: v4, v7, or v13? Justify your answer.

8. In the network below, vertex color indicates the group to which a vertex is affiliated. When vertex v7 wants to reduce the number of relations which he is maintaining, which relation would you advise him to end?

9. Count each brokerage role which vertex v7 may play in the network of Exercise 8.

7.8 Assignment

In our discussion of the triad, we compared complete and nearly complete triads. We stressed the opportunities which the incomplete structure offers to the person in the middle and the constraint exercised by the complete triad. Especially in the case of public behavior, that is, behavior which cannot be concealed from people in other groups, it has been argued that membership in several different cliques is very stressful because it obliges a person to conform to the - supposedly different - sets of norms of the different cliques. In this position, a person has very little room to maneuver.

In particular instances, this hypothesis contradicts the structural holes argument. If a person is a member in several different cliques, there are structural holes between the cliques, which may be exploited. In the network depicted in Figure 12, for example, vertex v3 may exploit structural holes between the other members in the 3-clique (v1 and v2) and the other members in the 4-clique (v4, v5, and v6). According to the structural holes hypothesis, vertex v3 is least constrained. According to the hypothesis of overlapping cliques, however, v3 is most constrained because it is a member in two cliques.
Having two competing hypotheses, it is interesting to see which one applies in a particular situation. Our case is a small hi-tech computer firm which sells, installs, and maintains computer systems. The data file hi-tech.net contains the friendship relations among the employees, which were gathered by means of the question: Who do you consider to be a personal friend? Three employees (Fran, Quincy, and York) did not return the questionnaire. Note that most friendship nominations are reciprocated, but not all (112 out of 147).

Some months later, employees tried to unionize the firm: they sought support among the employees to let the union have a say in the firm. The three top managers (class three in the partition hi-tech.union.clu) and three employees who were not directly involved (class two) were opposed to union certification of the firm. Five employees (class one) were pro-union, but two of them (Chris and Ovid) did not actively advocate the pro-union position. At the end, the proposal to unionize the firm was voted down. Chris resigned from the firm ten days before the vote because he did not want to participate in it. He rejoined the firm two days after the vote.

Analyze these data, which are joined in the project file Hi-tech.paj, and argue whether they support the structural holes argument or the overlapping cliques hypothesis. For the analysis of cliques, review Section 3.6 in Chapter 3. Pay attention to the position and behavior of Chris in particular. In addition, analyze the brokerage roles if the groups are defined by their stance towards unionization (hi-tech.union.clu) and find out whether this explains Chris’ behavior.

7.9 Further Reading

- The example is taken from J.H. Michael, ‘Labor dispute reconciliation in a forest products manufacturing facility’ (Forest Products Journal, 47 (1997), 41-45).
• Mark Granovetter’s *Getting a job: a study of contacts and careers* (Chicago: The University of Chicago Press, 1974) is the source of the strength of weak ties hypothesis. The second edition (1995) includes an appendix which surveys and analyses research based on this hypothesis.

• The theory of structural holes was introduced by R.S. Burt in his book *Structural holes. The social structure of competition* (Cambridge/London: Harvard University Press, 1992), which contains applications of this theory to careers of managers and to the profitability of business sectors.


• The example used in the assignment as well as the theory of constraint by overlapping cliques stems from D. Krackhardt’s ‘The ties that torture: Simmelian tie analysis in organizations’ (*Research in the Sociology of Organizations*, 16 (1999), 183-210).

### 7.10 Answers

1. The lines between v13 and v2, v14, and v15 are bridges. If you remove one of these lines, v2, v14, or v15 becomes an isolate, so the network contains two components instead of one.

2. Answer c is correct. The network contains three cut-vertices: v4, v7, and v13. Removal of any of these vertices disconnects the network. Cut-vertex v4 belongs to two bi-components: one bi-component contains vertices v1, v3, v4, v12, and v16 and the other bi-component consists of v4, v7, and v13. In addition, cut-vertex v7 belongs to the bi-component v5, v6, v7, v8, v9, v10, v11. Cut-vertex v13 belongs to three bi-components in addition to the ones already mentioned, because the three pairs of vertices connected by bridges (see Exercise 1) are bi-components.

3. In order to reduce the control of a cut-vertex, you must join two or more bi-components into one bi-component. A tie between members of the two largest bi-components, which are not cut-vertices, produces this effect, for instance, between vertices v3 and v6. Now, v4 and v7 are not cut-vertices anymore.

4. Statement a is correct. A vertex which is incident to a bridge can only have less than two neighbors if its only neighbor lies at the other side of the bridge. The vertex, then, is a ‘hanger’ like v2 in the network of Exercise 1. If you remove this vertex, you do not create a new component. If the vertex has two or more neighbors, however, it mediates between the vertex at the other side of the bridge and its other neighbor(s). When you remove the vertex, the latter neighbor is disconnected from the network, so the number of components in the network increases.

Statement b is not correct because a vertex in a bi-component may well be a
cut-vertex in the network at large, e.g., vertices v4, v7, and v13 in the network of Exercise 1. Statement c is not correct because a subnetwork is not necessarily connected. It may consist of several components which are not one bi-component by definition. Finally, statement d is not correct because the two paths between a pair of vertices may share a vertex in between the end-points, for instance, the paths v16-v3-v4-v7 and v16-v1-v4-v13-v7 in the network of Exercise 1. This vertex (v4) is a cut-vertex, so the component is not a bi-component.

5 It is easy to see that a pair of vertices connected by a bridge make up a bi-component of size two, since a bridge has two vertices (hence, size two), neither of which is a cut-vertex because if you remove one vertex, the remaining vertex still constitutes one component. Removing a vertex does not increase the number of components, so it is a bi-component by definition. Note that we look at the isolated bridge now, not at the entire network which may contain this bridge.

Does every bi-component of size two include a bridge in a simple undirected network? A bi-component of size two contains two vertices which are connected by lines because a bi-component is a component: a maximal connected subnetwork. It is important to note, that the two vertices are not connected indirectly, via one or more other vertices, because those vertices would also be part of the bi-component: they would constitute a cycle in an undirected network, which does not contain a cut-vertex since there are two paths connecting each pair of vertices. Hence, the two vertices are connected only by direct ties. If the network is simple, it does not contain multiple lines, so the two vertices are connected by one line, which is a bridge by definition because its removal disconnects the two vertices, which splits the original component in two, so the number of components increases.

6 Answer c is correct. It is not ruled out that a strong family tie is a bridge in an information network, for instance the tie between Frank and Gill in the strike network, so answer a is not correct. Strong and weak ties are defined on the basis of their frequency or intensity, not on their structural features, so it is not ruled out that strong ties occur outside cliques and weak ties occur inside cliques, so answers b and d are incorrect. However, there is a statistical association between the property (strength) of a tie and its structural location, so we may say that weak ties are more likely to be bridges in general, which is answer c.

7 Vertex v13 is less constrained than vertices v4 and v7. Note, first, that these three vertices have the same degree: each has five neighbors. As a consequence, the proportional strength of their relations is equal, namely 0.20. Therefore, we do not have to bother with the proportional strength of relations and we can simply count the number of structural holes around each vertex to find out who is least constrained. We count only one direct tie among the five neighbors of vertex v13, namely between v4 and v7, so nine out of the ten possible pairs of neighbors are not directly linked: they are
separated by structural holes. Among the neighbors of v7, two pairs are directly linked, which leaves eight structural holes. Finally, three pairs of v4’s neighbors are directly linked, so there are seven structural holes. More structural holes means less constraint, so vertex v13 is least constrained.

It is not wise to withdraw from relations which are part of a complete triad because that allows a neighbor to play the tertius strategy against you. Vertex v7 has only one tie outside a complete triad, namely the relation with vertex v10. He may safely withdraw from this relation because he is already connected to vertex v9, which is the most central member of the green group.

Vertex v7 does not broker between members of its own group, so he cannot play the coordinator role. V7 is connected to three vertices in the ‘green’ group, two of whom are directly connected, so he is an itinerant broker between v10 on the one hand and v6 and v9 on the other hand. V7 may mediate between v4, who is a member of his group, and three vertices in the ‘green’ group: v6, v9, and v10. Here, v7 plays the representative or gatekeeper role, which occurs three times. Finally, v7 may mediate between three green neighbors and one yellow neighbor, which yields six liaisons. Vertex v7 is a liaison predominantly.