# Graph and Digraph Glossary

A | B | C | D | E | F | G | H | I-J | K | L | M | N | O | P-Q | R | S | T | U | V | W-Z

## Acyclic Graph

A graph is acyclic if it contains no cycles.

## Adjacency Matrix

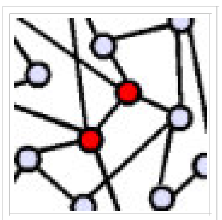
A 0-1 square matrix whose rows and columns are indexed by the vertices. A 1 in the ij-th position of the matrix means that there is an edge (or arc) from vertex i to vertex j. A 0 indicates that there is no such edge (or arc). Can be used for both graphs and digraphs.

## Adjacency Structure

A representation of a graph or digraph which lists, for each vertex, all the vertices that are adjacent to the given vertex.

## Adjacent

Two vertices are adjacent if they are connected by an edge. We often call these two vertices *neighbors*. Two adjacent vertices:



Two edges are adjacent if they have a vertex in common.

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# Ancestor

In a rooted tree, a vertex on the path from the root to the vertex. Vertex v is an ancestor of vertex w if and only if w is a descendant of v.

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# Arc

A directed edge of a digraph. Some authors use it as a synonym for an edge of a graph. Other synonyms for arc in a digraph are arrow, directed line, directed edge, and directed link.

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# Arc List

A representation of a digraph using the arcs of the digraph. Can be an unordered listing of the ordered pairs, or a pair of ordered lists with the starting vertex in one list and the ending vertex in the corresponding position of the second list.

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# Bipartite Graph

A graph is bipartite if the vertices can be partitioned into two sets, X and Y, so that the only edges of the graph are between the vertices in X and the vertices in Y. Trees are examples of bipartite graphs. If G is bipartite, it is usually denoted by G = (X, Y, E), where E is the edge set.

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# Binary Code

An assignment of symbols or other meanings to a set of bitstrings.

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# Binary Search Tree

A binary tree that has been labelled with numbers so that the right offspring and all of its descendants have labels smaller than the label of the vertex, and the left offspring and all its descendants have labels larger than that of the vertex. .

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# Bridge

An edge in a graph whose removal (leaving the vertices) results in a disconnected graph. Also known as a *cut-edge*.

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# Child

In a rooted tree, a vertex v is a child of vertex w if v immediately succeeds w on the path from the root to v. Vertex v is a child of w if and only if w is the parent of v.

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# Chromatic Number

The chromatic number of a graph is the smallest k for which the graph is k-colorable. The chromatic number of the graph G is denoted by *X*(G). [*X* is the greek letter chi].

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# Clique

A subgraph that is a complete graph.

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# Closure

The closure of a graph G with n vertices, denoted by c(G), is the graph obtained from G by repeatedly adding edges between non-adjacent vertices whose degrees sum to at least n, until this can no longer be done. Several results concerning the existence of hamiltonian circuits refer to the closure of a graph.

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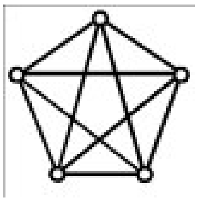
# Complete

A complete graph is a simple graph in which all pairs of vertices are adjacent. They are denoted by $K_n$, where n is the number of vertices. (The K is in honor of Kuratowski, a pioneer in graph theory.) The corresponding concept for digraphs is called a complete symmetric digraph, in which every **ordered** pair of vertices are joined by an arc. Here is the complete graph on five vertices, $K_5$:



# Connected Component

In a graph, a (connected) component is a maximal, connected, induced subgraph. Maximal means that there is no larger connected, induced subgraph containing the vertices of the component.

# Condensed Graph

Given a graph G, if two vertices of G are identified and any loops or multiple edges created by this identification removed, the resulting graph is called the *condensed graph*.

# Connected

A connected graph is one in which every pair of vertices are joined by a walk. A graph which is not connected is called *disconnected*, and breaks up into connected components.

# Cycle

A closed path with at least one edge.

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# Decision Tree

A binary tree used to represent an algorithm for sorting by comparisons. The leaves of the tree represent the possible outcomes (orderings), while the other vertices represent test questions which have a yes or no answer.

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# Degree

The degree of a vertex is the the number of proper edges incident with the vertex plus twice the number of self-loops at the vertex. The degree of a graph is the maximum degree of all of its vertices.

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# Degree Sequence

The degree sequence of a graph is the sequence formed by arranging the vertex degrees in non-decreasing order.

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# Descendant

In a rooted tree, a descendant of vertex v is any vertex w whose path from the root contains v.

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# Diameter

The diameter of a graph is the length of the longest walk you are forced to use to get from one vertex to another in that graph. You can find the diameter of a graph by finding the distance between every pair of vertices and taking the maximum of those distances.

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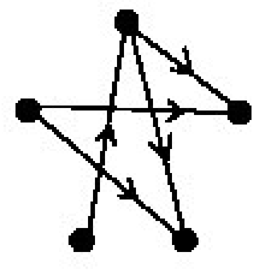
# Digraph

A digraph is a graph in which the edges are directed and called arcs. More formally, a digraph is a set of vertices together with a set of ordered pairs of the vertices, called arcs. Here is a digraph on 5 vertices:

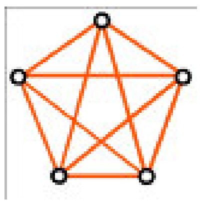


# Distance

The distance between two vertices is the length of the shortest walk between them.

# Edge

An edge connects two vertices in a graph. We call those two vertices the endpoints of the edge. Other synonyms for edge are arc, link and line. Here are the edges of a graph (in red):



# Forest

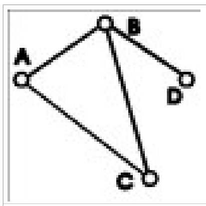
A graph which contains no cycles. The connected components of a forest are trees.

# Graph

A graph is basically a collection of dots, with some pairs of dots being connected by lines. The dots are called vertices, and the lines are called edges.

More formally, a graph is two sets. The first set is the set of vertices. The second set is the set of edges. The vertex set is just a collection of the labels for the vertices, a way to tell one vertex from another. The edge set is made up of unordered pairs of vertex labels from the vertex set.

Here is a diagram of a graph, and the sets that the graph is made from:

| | |
|---|---|
|   | V={A,B,C,D}<br>--The vertex set.<br>E={(A,B) , (A,C) , (B,C) , (B,D)}<br>--The edge set. |
| A graph diagram. | The sets that make up a graph. |

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# Hamiltonian

A walk or circuit in a graph is said to be *hamiltonian* if each vertex of the graph appears in it precisely once. Paths and cycles of digraphs are called hamiltonian if the same condition holds. A graph containing a hamiltonian circuit, or a digraph containing a hamiltonian cycle is referred to as a *hamiltonian graph or digraph*.

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# Height

The height of a rooted tree is the length of the longest path starting at the root of the tree.

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# Homeomorphic

Two graphs are homeomorphic if they can both be obtained from a common graph by a sequence of replacing edges by simple chains. In appearance, homeomorphic graphs look like ones that have extra vertices added to or removed from edges.

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# Incidence Matrix

A 0-1 matrix whose rows are indexed by the vertices of a graph and whose columns are indexed by the edges. A 1 in the ij-th position of the matrix means that the vertex i is on the edge j. A 0 indicates that it is not. In some treatments, a self-loop at vertex i is indicated by a 2 in the ii-th position.

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# Internal Vertex

A vertex in a tree which is not a pendant vertex.

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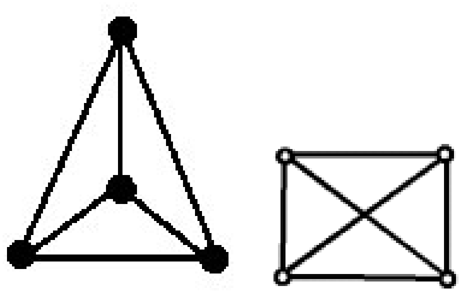
# Isomorphic

Two graphs are isomorphic if they are they same graphs, drawn differently. Two graphs are isomorphic if you can label both graphs with the same labels so that every vertex has exactly the same neighbors in both graphs. Here are two isomporphic graphs:



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# k-Colorable

A graph is said to be k-colorable if each of its vertices can be assigned one of k colors in such a way that no two adjacent vertices are assigned the same color. The assignment is called a *coloring*.

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# Label

Labels are just the names we give vertices and edges so we can tell them apart. Usually, we use the integers 1, 2, ..., n as the labels of a graph or digraph with n vertices. The assignment of label to vertex is arbitrary.

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# Leaf

A vertex of degree 1. Also known as a pendant vertex.

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# Level

In a rooted tree, the vertices at the same distance from the root are said to be at the same *level*. The root is considered to be at level 0 and the height of the tree is the maximum level.

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# Loop

An edge or arc from a vertex to itself is called a *loop*. Loops are not allowed in simple graphs or digraphs. Also called *self-loops*.

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# m-ary Tree

A rooted tree in which every vertex has m or fewer offspring. When m = 2, these are called *binary trees*. An m-ary tree is ***complete*** if every internal vertex has exactly m children, and all leaves have the same depth.

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# Matching

A matching in a graph is a set of edges such that every vertex of the graph is on at most one edge in the set.

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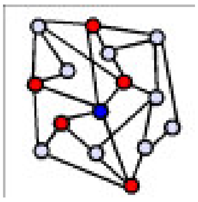
# Neighborhood

The neighborhood of a vertex is all the vertices that it is adjacent to (all of the vertex's neighbors). Here we have a vertex (in blue) and the vertices in its neighborhood (in red):



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# Node

Another word for vertex.

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# Offspring

In a rooted tree, the vertices adjacent to a given vertex at the next higher level are called the *offspring* of the given vertex. They are sometimes called *children*. The *descendents* of a vertex are the vertices in the set of vertices which are offspring, or offspring of offspring, etc. of the given vertex..

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# Order

The order of a graph is the number of vertices it has.

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# Ordered Tree

A rooted tree in which the children of each vertex are assigned a fixed ordering.

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# Orientation

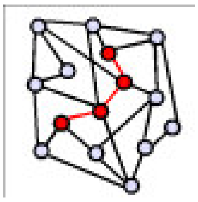
An assignment of a direction to each edge of a graph. A graph which has been given an orientation is called an *oriented graph*, and is a digraph.

# Parent

In a rooted tree, vertex w is the parent of vertex v if w immediately precedes v on the path from the root to v. Vertex w is the parent of v if and only if v is a child of w.

# Path

A path is a trail with no repeated vertices (except possibly the first and last). Here is an example of a path:



# Pendant Vertex

A vertex of degree 1. Also known as a leaf.

# Prefix Code

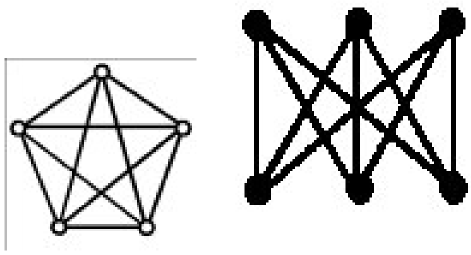
A binary code with the property that no codeword is an initial substring of any other codeword.

# Perfect Matching

In a graph with 2n vertices, a matching with n edges is said to be perfect. Every vertex of the graph is saturated by a perfect matching. Another term for a perfect matching is a *1-factor*.

# Planar

A planar graph is a graph that you can draw on a flat surface, or plane, without any of the edges crossing. Graphs that cannot be drawn on the plane without crossed edges are called non-planar graphs. Any graph that has either of the following graphs as subgraphs are non-planar:



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# Reduced Graph

If an edge, a, is removed from a given graph G, the resulting graph, denoted $G'_a$ is referred to as a *reduced graph*.

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# Regular

In a regular graph, each vertex has the same degree. If this common degree is k, then we say that the graph is *k-regular*.

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# Rooted Tree

A tree in which one vertex has been distinguished. The distinguished vertex is called the *root* of the tree. If the tree is directed, there is a directed path from the root to each vertex of the tree.

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# Saturated vertex

A vertex in a graph which is on an edge of a matching is said to be *saturated*. Given a matching M, if X is a set of vertices saturated by M, then M is said to be an

*X-saturating matching*.

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# Sibling

Two vertices in a rooted tree are siblings if they have the same parent.

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# Size

The size of a graph is the number of edges it has.

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# Spanning Subgraph

A subgraph of the graph G which contains all of the vertices of G.

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# Spanning Tree

A spanning subgraph of a graph which is also a tree.

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# Strongly Connected

In a digraph there are many degrees of connectedness. A strongly connected digraph is one in which any vertex can be reached from any other vertex by a directed walk.

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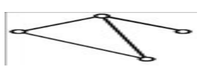
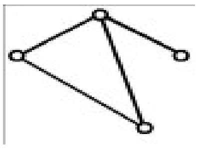
# Subgraph

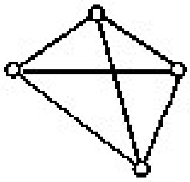
A subgraph of a graph is some smaller portion of that graph. Here is an example of a subgraph:



A graph A subgraph

An *induced (generated) subgraph* is a subset of the vertices of the graph together with **all** the edges of the graph between the vertices of this subset. The induced subgraph of the above example is:



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# Topological Order

A topological ordering of a digraph is a labelling of the vertices with consecutive integers so that every arc is directed from a smaller label to a larger label.

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# Tournament

A tournament is a digraph in which there is exactly one arc between any two vertices. A tournament is said to be *transitive* if whenever (a,b) and (b,c) are arcs of the tournament, then (a,c) is also an arc.

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# Trail

In a graph, a trail is a walk with no repeated edges.

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# Tree

A connected graph containing no cycles.

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# Underlying Graph

The graph that results from removing the directions on all the arcs of a digraph or partially directed graph.

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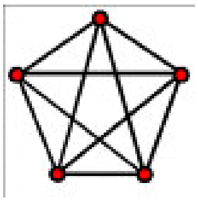
# Vertex

A vertex is a 'dot' in a graph. The plural of vertex is 'vertices', as in, 'this graph has five vertices'. Other synonyms for vertex are node, point or state. Here are the vertices of a graph (in red):



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# Walk

In a graph, a walk from vertex $v_0$ to vertex $v_n$ is an alternating sequence

$$W = <v_0,e_1,v_1,e_2,...,v_{n-1},e_n,v_n>$$

of vertices and edges, such that the endpoints of edge $e_i$ are $v_{i-1}$ and $v_i$, for i = 1,...,n. The *length* of a walk is the number of edges in it. A walk is *closed* if $v_0 = v_n$ and *open* otherwise.

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