# Layouts for GD01 Graph-Drawing Competition

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# How layouts of graphs were obtained

All work was done using our program package **`Pajek`**, which is freely available at:

`http://vlado.fmf.uni-lj.si/pub/networks/pajek/`

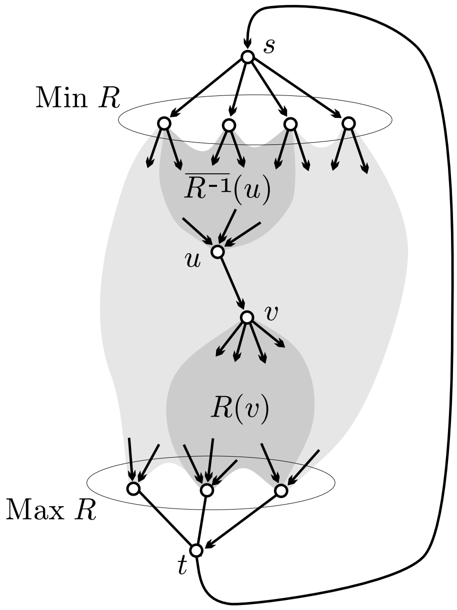
## Graph A

### Citation networks analysis

In a given set of units $E$ (articles, books, works, …) we introduce a $citing$ relation $R \subseteq E \times E$

$$xRy \equiv y \text{ cites } x$$

which determines a *citation network* $(E, R)$. A citing relation is usually *irreflexive*, $\forall x \in E : \neg xRx$, and (almost) *acyclic*, $\forall x \in E\, \forall k \in \mathbb{N}^+ : \neg xR^k x$. The citation network is *standardized* by adding, if necessary, artificial *source* vertex $s$ and *sink* or *terminal* vertex $t$ and the arc $(t, s)$ (see figure).




An approach to the analysis of citation network is to determine for each unit / arc its *importance* or *weight*. These values are used afterwards to determine the essential substructures in the network. Hummon and Doreian (1989, 1990) proposed three methods of assigning weights $w : R \to \mathbb{R}_0^+$ to arcs:

- NPCC method: $w_1(u, v) = |\overline{R^{-1}}(u)| \cdot |\overline{R}(v)|$
- Paths count method: $w_2(u, v) = N(u, v)$, where $N(u, v)$ denotes the number of different paths from $\mathrm{Min}\, R$ to $\mathrm{Max}\, R$ (or from $s$ to $t$) through the arc $(u, v)$
- SPLC method: $w_3(u, v) = N'(u, v)$, where $N'(u, v)$ equals to $N(u, v)$ from paths count method over the network $(E, R')$, $R' := (R \cup \{s\}) \times (E \setminus \{s, t\})$

The last two methods are efficiently (Batagelj, 1991, 1994) implemented in Pajek and can be applied also on (very) large acyclic networks. Let $N^-(v)$ denotes the number of different paths from $s$ to $v$, and $N^+(v)$ denotes the number of different paths from $v$ to $t$. Then $N(u,v) = N^-(u) \cdot N^+(v),\ (u,v) \in R$.

$N$ and $N'$ are flows in the network since they obey the *Kirchoff's node law*:

For every node $v$ in a citation network $(E, R)$ in standard form it holds

$$\text{incoming flow} = \text{outgoing flow}$$

Therefore the *total flow* through the citation network equals $N(t,s)$. This gives us a natural way to normalize the weights

$$w(u,v) = \frac{N(u,v)}{N(t,s)} \quad \Rightarrow \quad 0 \leq w(u,v) \leq 1$$

If $C$ is a minimal cut-set it also holds

$$\sum_{(u,v)\in C} w(u,v) = 1$$

We can assign weights also to vertices

$$w(v) = \frac{N^-(v) \cdot N^+(v)}{N(t,s)}$$

## References


1. Batagelj V.: An Efficient Algorithm for Citation Networks Analysis. Presented at EASST'94, Budapest, Hungary, August 28-31, 1994. First presented at the Seminar on social networks. University of Pittsburgh, January 1991.

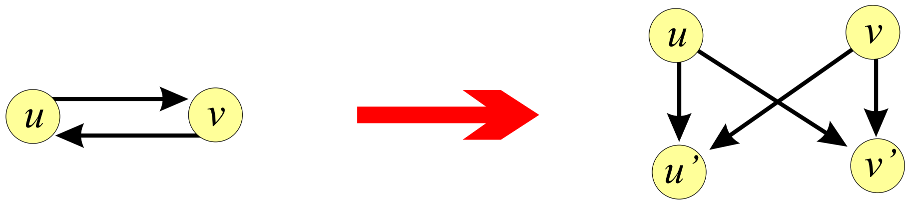
2. Hummon N.P., Doreian P.: Connectivity in a Citation Network: The Development of DNA Theory. Social Networks, **11**(1989) 39-63.

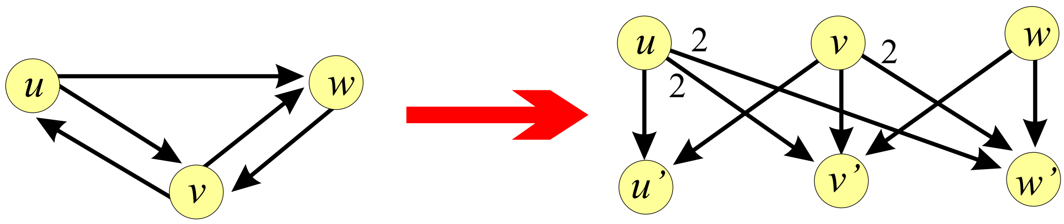
3. Hummon N.P., Doreian P.: Computational Methods for Social Network Analysis. Social Networks, **12**(1990) 273-288.

4. Hummon N.P., Doreian P., Freeman L.C.: Analyzing the Structure of the Centrality-Productivity Literature Created Between 1948 and 1979. Knowledge: Creation, Diffusion, Utilization, **11**(1990)4, 459-480.


## Layouts of graph A

In graph A the relation is the reverse of citing relation. The original graph A has 311 vertices. It has 6 weak components. Searching for strong components (testing acyclicity) it turns out that the graph A is not acyclic. A large strong component was generated by an erroneous arc ( GD94/143 Eades, GD98/423 Eades). After reversing it 4 small strong components remained, corresponding to mutual references { GD94/286 Garg, GD94/298 Papakostas }, { GD94/328 Di Battista, GD94/340 Bose, GD94/352 ElGindy }, { GD95/8 Alt, GD95/234 Fekete } and { GD95/140 Chandramouli, GD95/300 Heath } . To obtain an acyclic graph, required by citations analysis method, we applied the following 'preprint' transformation:




Each paper from a strong component is duplicated with its 'preprint' version. The papers inside strong component cite preprints.

The pictures were exported as nested partitions into SVG format that allows interactive display of different *slices* – subgraphs induced by arcs with weights larger than a threshold value. These pictures are available at `http://vlado.fmf.uni-lj.si/pub/GD/GD01.htm`. In the paper form we can present only some snapshots.

The first picture (see Figure 1) displays complete network after 'preprint' transformations (320 vertices). The vertices are put in layers (vertical position and color of vertices) according to the years of publication. The placement of vertices inside the layer was determined by local optimization. The width and the color density of an arc and the size of a vertex are proportional to their citation weights.

The second and the third picture display the main parts (slices) of the citation network at threshold values 0.02 (Figure 2) and 0.05 (Figure 3). The red arcs belong to the 'main path'.

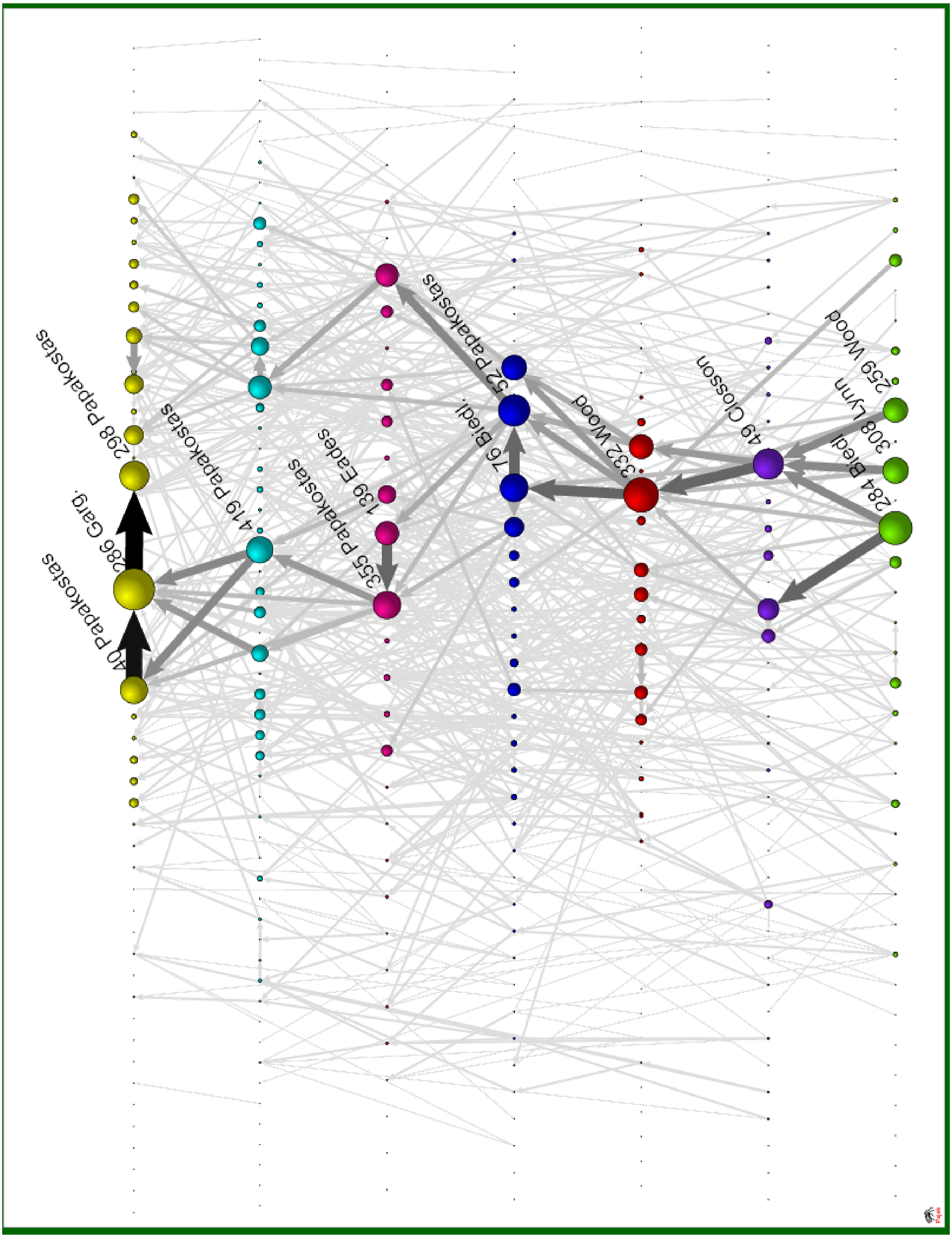



Figure 1: Graph A – complete graph.

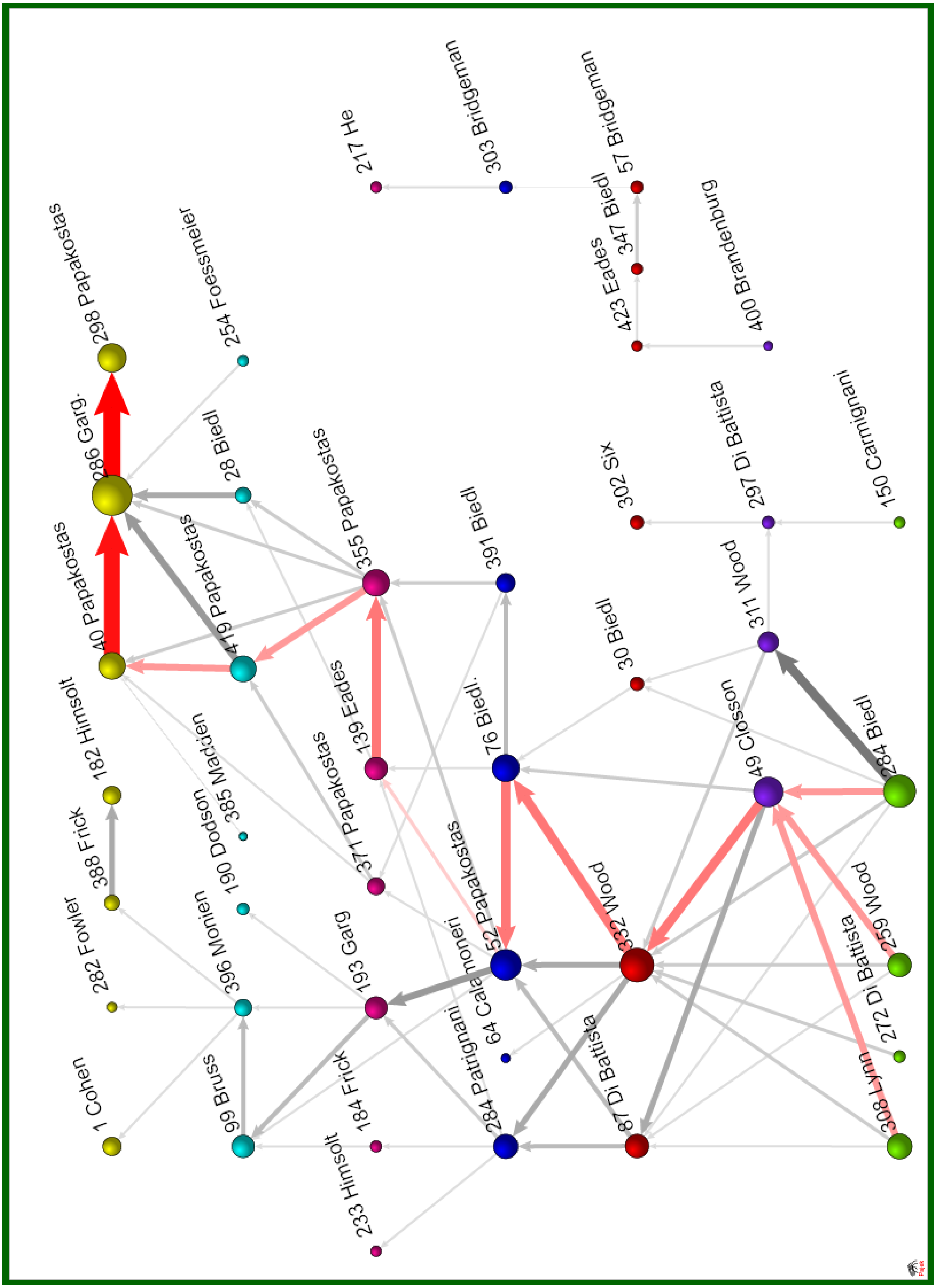



Figure 2: Graph A – level 0.02.

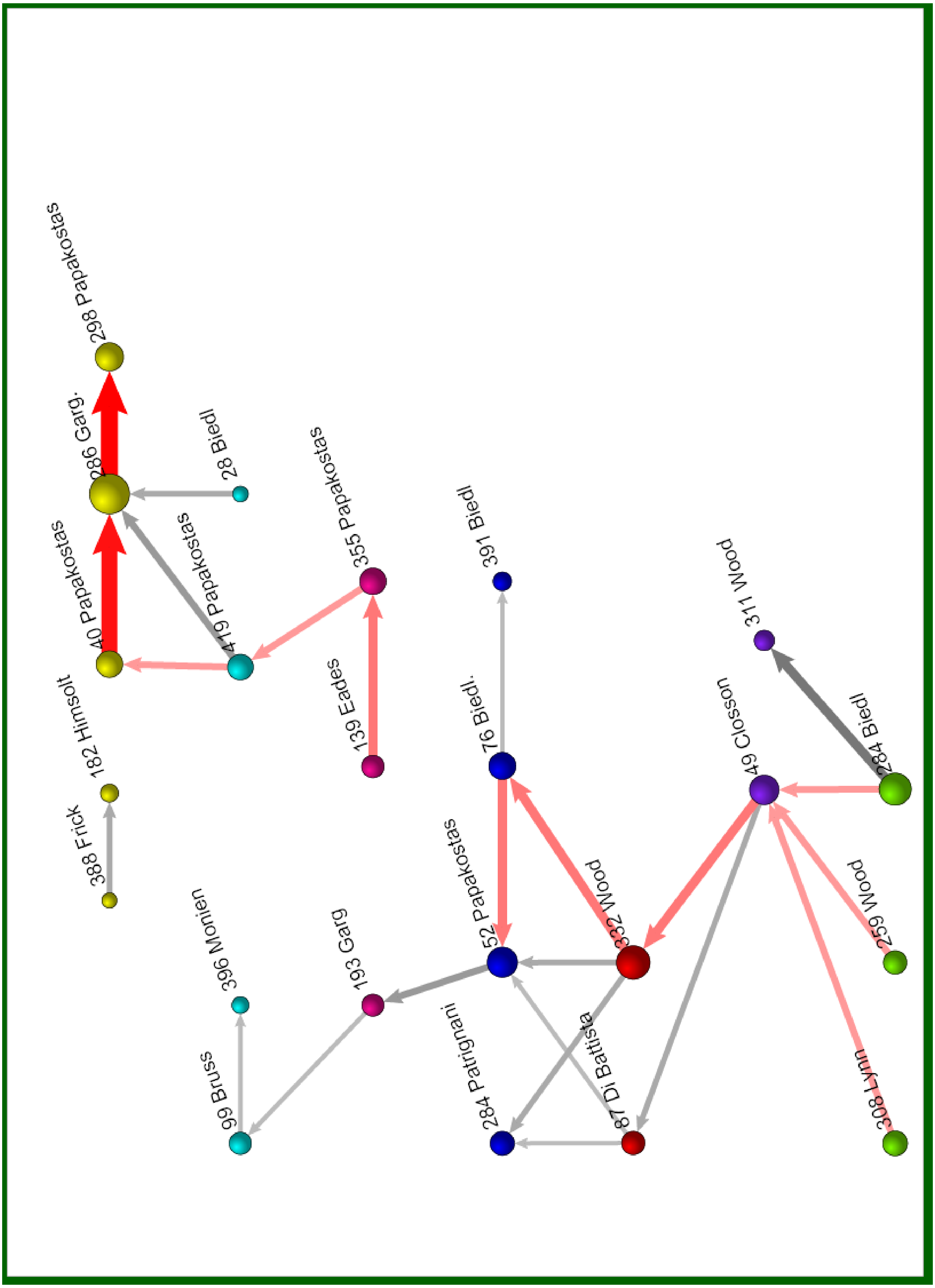



Figure 3: Graph A – level 0.05.

## Graph B

To obtain the 'central symmetric' picture of graph B energy drawing was used, followed by manual grid positioning of vertices. To save the space the lower part of the picture (Figure 4) was manually mirrored across the vertical axis (Figure 5).

## Graph C

Graph C is an acyclic directed graph. Such graphs can be topologically sorted. The corresponding adjacency matrix has zero lower triangle and diagonal. Since the graph is rather dense we decided to use the matrix representation to visualize the graph structure (Figure 6). Layers are represented by blocks divided by blue lines.

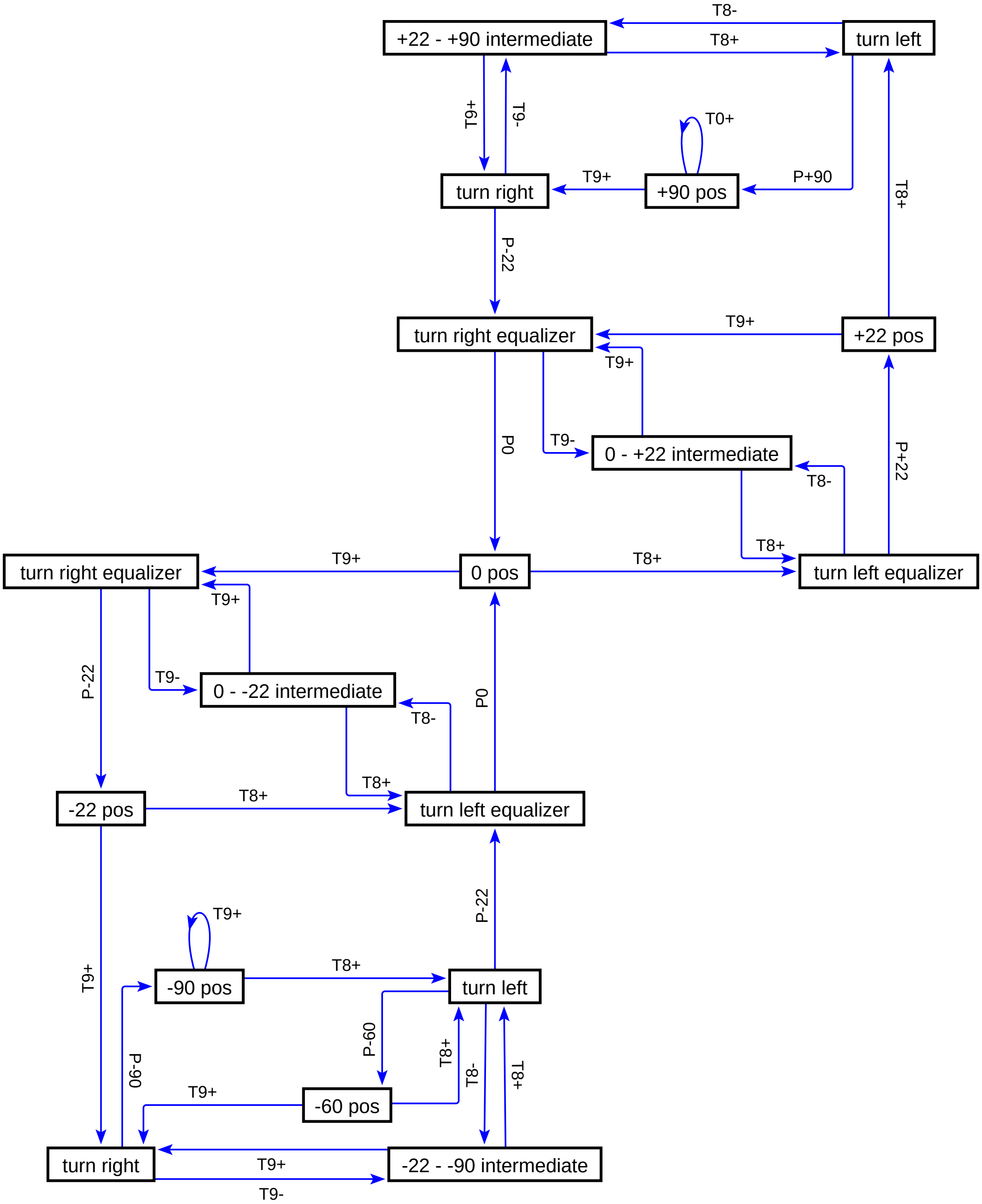



Figure 4: Graph B – ’central symmetric’.

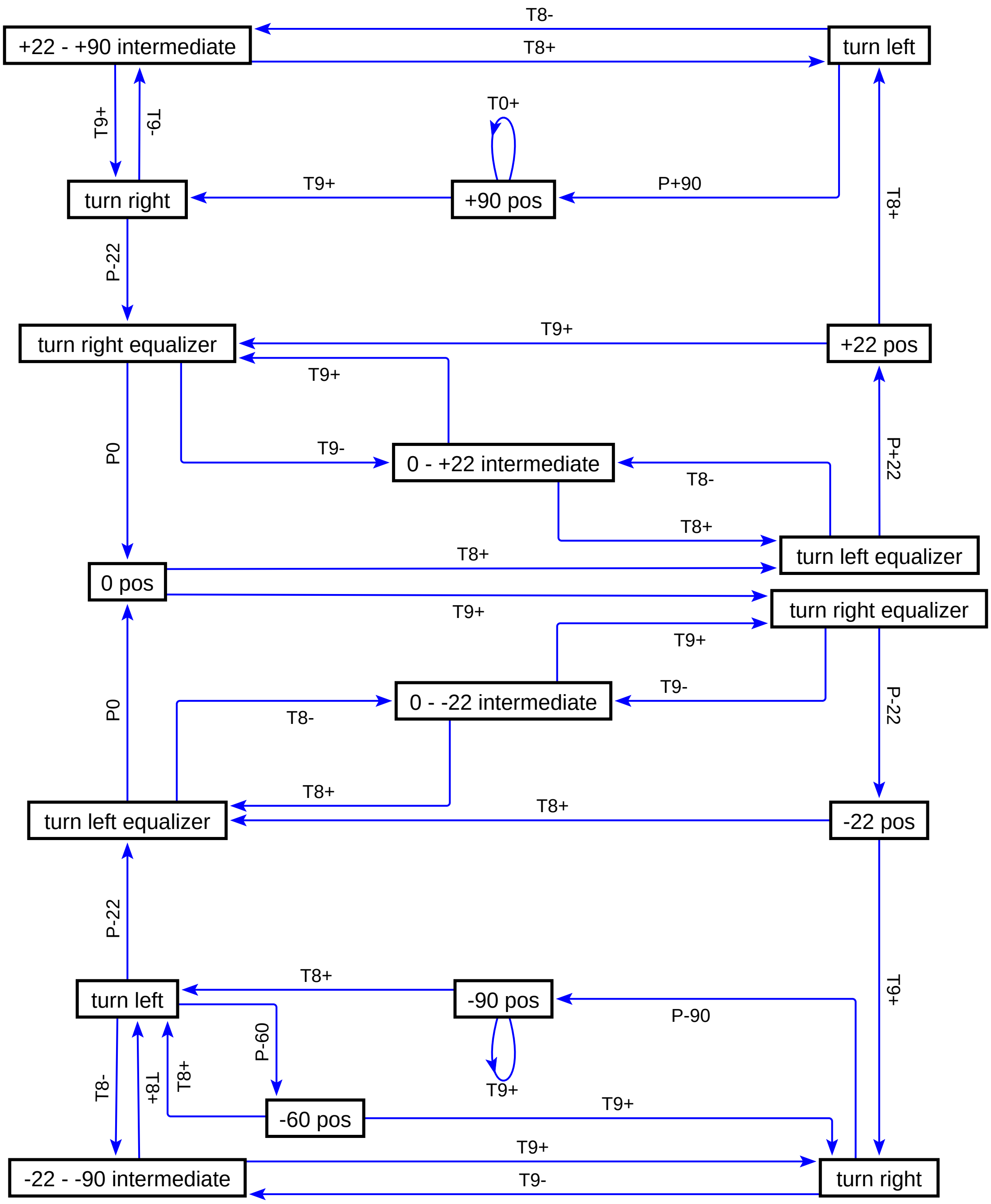



Figure 5: Graph B – mirror.

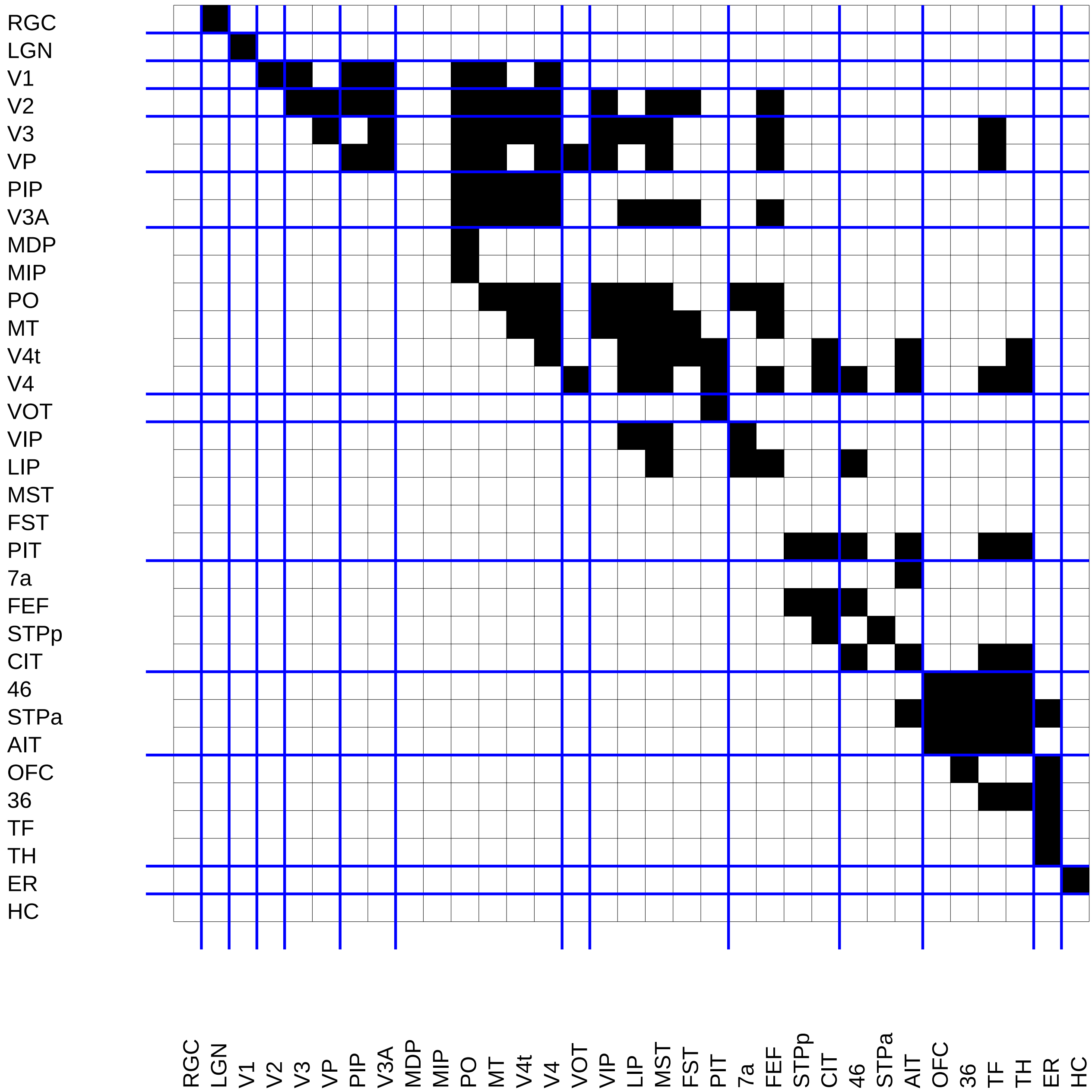



Figure 6: Graph C – matrix representation.