Ruin probability and risky investments

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Abstract

In this presentation we deal with ruin probability of an insurance company. If we denote risk reserve process with $(X_t)_{t\geq 0}$, then the probability of ruin is the probability of event $\{X_t < 0 \text{ for some } t > 0\}$. In the classical Cramér–Lundberg model, the claim number process is actually Poisson process, and the claim sizes are positive iid random variables. Under the assumptions of net profit condition and the existance of Lundberg coefficient, the ruin probability as a function of the initial capital u decreases exponentially.

It is natural to study ruin problem in application to an insurance company which continuously invests its capital in a risky asset. We assume that the price of this risky asset follows a geometric Brownian motion with mean return $a > 0$ and volatility $\sigma > 0$, which satisfies the following stochastic differential equation

$$dV_t = V_t(adt + \sigma dW_t)$$

($(W_t)_{t\geq 0}$ is a standard Brownian motion). Now we expect ruin probability to be greater than in the classical Cramér–Lundberg model. Indeed, in the case of small volatility, i.e. $a - \sigma^2/2 > 0$, we find exact the asymptotic upper and lower bounds for the ruin probability $\Psi(u)$, as the initial capital u tends to infinity. We show that, for sufficiently large u, $\Psi(u)$ is bounded by power functions with the same exponent $\beta := 2a/\sigma^2 - 1$. In the special case of exponential premium rate, we derive the exact asymptotics for the ruin probability. Therefore, we conclude that the ruin probability is no more exponential but a power function of the initial capital. In the case of large volatility, i.e. $a - \sigma^2/2 \leq 0$, we show that ruin probability equals 1 for any $u \geq 0$. We support some of these results by simulations.

This presentation is based on the paper of S. Pergamenshchikov and O. Zeitouny, Ruin probability in the presence of risky investments.